Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

and help fix the principles according to which it works. Let us take one of the equations used in calibrating the balance. The equation[1]

$$x^3 + 3x^2 - 2x - 5 = 0$$

has a root between 1 and 2. Correct to 3 decimal places it is 1.330. This is outside the range of the balance. Form a new equation whose roots are half those of the original. The transformed equation is

$$8x^3 + 12x^2 - 4x - 5 = 0.$$

The root now sought is 0.665. Place the balance in equilibrium with buckets empty. The buckets on the curves $y = \pm x^4$ are left empty. In the bucket on $y = x^3$ (right) are placed 8 of the ball bearings. Similarly $y = x^2$ (right) is loaded with 12. The line $y = -x$ (left) is loaded with 4, and 5 ball bearings are placed in the bucket at the extremity of the left arm of the balance. Equilibrium is again established by means of the guide which is used to move the stirrups supporting the buckets to their proper position. The cross-hair should then be over 0.665 on the scale, the third figure being estimated. Obviously, the root of the original equation is 1.330.

The balance has stood the test of the class-room for a year and can be depended upon for roots correct to two decimal places, with a close approximation to the third. But more than for its worth in solving equations is it to be valued as an aid in fixing in the students' minds quite a number of the fundamental principles in the theory of equations.

GROUPS OF THE FIGURES OF ELEMENTARY GEOMETRY.

By G. A. MILLER, University of Illinois.

The groups of movements of figures of elementary geometry were considered in an article published in this MONTHLY, volume 10 (1903), page 214. The present note is complementary to this article, but the considerations are based upon entirely different principles. In fact, in the present note we inquire into the substitution group, on the sides or the edges of a figure, which transform this figure into others having the same absolute area or the same absolute volume; while the earlier considerations related to those movements which transform the figure as a whole into itself, but interchange some of its parts.

Let P represent any convex plane polygon of n sides. By interchanging two adjacent sides of P without affecting the other sides we can obtain another convex polygon with the same n sides and with the same absolute area. By a succession of such interchanges it is clearly possible to effect a transposition of any two sides without affecting the absolute area. Since every substitution on the n sides is the product of transpositions, it results that the $n!$ polygons obtained from P in the given manner have the same absolute area, and are conjugate under the symmetric group on their sides.

---

[1] Fite, *College Algebra*, pp. 192–7.

A necessary and sufficient condition that one of these polygons has a maximal area is that all of them have a maximal area, and hence they can all be inscribed in the same circle, as is also otherwise evident. In fact, this is a direct consequence of the well-known theorem that the circle incloses a larger area than any other plane figure having the same perimeter, since it is possible to construct a convex polygon inscribed in a circle whose sides are equal, in order, to those of any given convex polygon. As a special case there results the elementary theorem that we may interchange, according to any substitution of the symmetric group of degree n, the n sides of any convex polygon inscribed in a circle without affecting the area of this polygon.

The tetrahedron having no two equal edges and no edge which is equal to the sum of two of its edges presents an interesting elementary example from the present point of view. It is evident that the three edges of the base may be interchanged cyclically provided the other three edges are interchanged in the same order. That is, the absolute volume of the tetrahedron will not be affected by eight substitutions of the form $abc \cdot def$, since any one of the four faces may be used as a base. Moreover, if all the edges of a face are left invariant every interchange of the other three edges will in general affect the volume. It will be assumed that the tetrahedron in question has this property.

Hence the substitution group, composed of all the substitutions on the six edges which transform this tetrahedron into another having the same absolute volume, cannot involve more than 24 different substitutions since no more than six such substitutions are possible on the edges of a face. It is very easy to see that there are actually 24 such substitutions. In fact, every face can evidently be transformed into the base, and the edges of the base can be transformed in six different ways without affecting the absolute volume.

In addition to the eight substitutions of the given form there are nine of the form $ab \cdot cd$, and six of the form $abcd \cdot ef$, in addition to the identity. Hence it is easy to identify the substitution group obtained in the given manner as the transitive group of order 24 and of degree 6 which is composed of positive substitutions. This group was denoted by Cayley by the symbol $(+\, abcdef)_{24}$, *Quarterly Journal of Mathematics,* volume 25 (1891), page 81. As is well known it is simply isomorphic with the symmetric group of degree 4, and hence its abstract properties may be directly deduced from those of the latter group.

As there are 720 substitutions on the six edges of the given tetrahedron, and as only 24 of these transform this tetrahedron into one having the same absolute volume, it is generally possible to construct 30 tetrahedrons, having the given six edges, such that no two have the same absolute volume. This fact can also be seen by observing that 20 different triangles can be constructed with these six lines as sides. If we take any one of these triangles as a base it is possible to arrange the other three lines in such a way as to obtain six different tetrahedrons. Hence there result 120 tetrahedrons which have equal absolute volumes, in sets of four, since each face can be used as a base. For greater details along this line the reader may consult an article by Karl Schwering, *Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht,* volume 43 (1912), page 409.